Stepped Frequency Range Profile Parameters

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1 Stepped Frequency Data

Suppose an ideal pulsed monochromatic complex sinusoidal signal, $a(t)\exp(j2\pi Ft)$ is transmitted and reflects from a point target at range R_0. The reflected signal is $a(t-2R_0/c)\exp(j2\pi F(t-2R_0/c))$. This echo signal is demodulated by mixing with the reference signal $\exp(-j2\pi Ft)$ and sampling at time $t = 2R_0/c$, giving the receiver output voltage (a complex scalar) $X = a(0)\exp(-j4\pi FR_0/c)$. Now suppose we repeat this measurement using a series of M stepped frequencies, $F_m = F_0 + m\Delta F$, $m = 0,1,\ldots,M-1$. Note that the total bandwidth of the date is $B = M\Delta F$ Hz. The series of receiver outputs as a function of the stepped frequency is

$$\begin{aligned} X[m] &= a(0)\exp\left[-j4\pi\left(F_0 + m\Delta F\right)R_0/c\right] \\ &= a(0)\exp\left[-j4\pi F_0 R_0/c\right]\exp\left[-j4\pi m\Delta F R_0/c\right] \\ &\equiv A\exp\left[-j4\pi m\Delta F R_0/c\right] \end{aligned} \tag{1}$$

$X[m]$ can be viewed as a set of uniform spectral samples. Consider the N-point inverse DFT of $X[m]$ ($N \geq M$; the upper limit on the summation is M instead of N because there are only M spectral samples):

$$\begin{aligned} x[n] &= \frac{1}{N}\sum_{k=0}^{M-1} X[km]e^{j2\pi nm/N} \\ &= \frac{A}{N}\sum_{m=0}^{M-1}\exp\left[j2\pi\left(\frac{n}{N} - \frac{2R_0\Delta F}{c}\right)m\right] \end{aligned} \tag{2}$$

Applying the geometric summation formula gives us a closed form result [2]:

$$x[n]=\frac{A}{N}\left\{\frac{1-\exp\left[j2\pi\left(\frac{n}{N}-\frac{2R_0\Delta F}{c}\right)M\right]}{1-\exp\left[j2\pi\left(\frac{n}{N}-\frac{2R_0\Delta F}{c}\right)\right]}\right\}$$

$$=\frac{A}{N}\exp\left[j\pi\left(\frac{n}{N}-\frac{2R_0\Delta F}{c}\right)(M-1)\right]\bullet$$

$$\bullet\left\{\frac{\exp\left[-j\pi\left(\frac{n}{N}-\frac{2R_0\Delta F}{c}\right)M\right]-\exp\left[j\pi\left(\frac{n}{N}-\frac{2R_0\Delta F}{c}\right)M\right]}{\exp\left[-j\pi\left(\frac{n}{N}-\frac{2R_0\Delta F}{c}\right)\right]-\exp\left[j\pi\left(\frac{n}{N}-\frac{2R_0\Delta F}{c}\right)\right]}\right\}$$

$$=\frac{A}{N}\exp\left[j\pi\left(\frac{n}{N}-\frac{2R_0\Delta F}{c}\right)(M-1)\right]\left\{\frac{\sin\left[\pi\left(\frac{n}{N}-\frac{2R_0\Delta F}{c}\right)M\right]}{\sin\left[\pi\left(\frac{n}{N}-\frac{2R_0\Delta F}{c}\right)\right]}\right\} \tag{3}$$

This familiar “digital sinc” or “asinc” response is the image of the point scatterer at range R_0.

The peak of the response, *i.e.* the sample number n_p corresponding to R_0, occurs when the argument of the sine functions equals zero, which occurs when

$$n_p=\frac{2R_0\Delta FN}{c} \tag{4}$$

Note that to increase n_p by 1, R_0 must increase by $c/2N\Delta F$ meters. Thus, the inverse DFT of the frequency stepped data produces data that is sampled in range at an interval of $c/2N\Delta F$ meters.

Since the N-point inverse DFT is periodic with period N samples (as can be seen from Eq. (2)), the range image generated by the inverse DFT will have an extent of N times the sample spacing before it repeats; thus the image extent in meters is

$$R_{\max}=N\left(\frac{c}{2N\Delta F}\right)=\frac{c}{2\Delta F} \tag{5}$$

There is no guarantee that n_p will be integer, or will fall within the inverse DFT index range of 0 to N-1. A non-integer value of n_p just means that the scatterer range does not fall exactly on one of the range sample points, thus the true peak of the response will be between range samples and a range “straddle loss” [1] will result. The actual value of n_p can be much larger than N-1. Consider a scenario where the scene size is 1 km, so that ΔF = 150 kHz (see Eqn. (5)); N = 256; and R_0 = 3500 m. In this case, n_p = 896. Because of the periodicity of the inverse DFT (with period N), the actual location of the peak output will be at

$$n_p = \frac{2R_0 \Delta F N}{c} \bmod N \tag{6}$$

which is n_p = 128 in this case. Put another way, the peak index corresponds to the range evaluated modulo $R_{\max}$. In this case, $R_{\max}$ = 1 km, so R_0mod($R_{\max}$) = 3500mod(1000) = 500 m. Using either (4) or (6), the resulting peak index is n_p = 128.

Next consider the Rayleigh (peak-to-null) resolution of the image of the scatterer. The first null of the response in Eq. (3) occurs when the argument of the sine function in the numerator equals π. This occurs at sample

$$n_z = \frac{N}{M} + \frac{2R_0 \Delta F N}{c} = n_p + \frac{N}{M} \tag{7}$$

The Rayleigh resolution is the spacing between n_z and n_p, and is thus (in samples and, using the sample spacing $c/2N\Delta F$ meters from above, in range)

$$\Delta n = \frac{N}{M} \quad \Rightarrow \quad \Delta R = \frac{c}{2M\,\Delta F} = \frac{c}{2B} \tag{8}$$

Thus the range resolution follows the usual $c/2B$ rule.

2 Range Frequency

Note that the range resolution ΔR is the inverse of the stepped frequency bandwidth, scaled to spatial frequency units by the factor $2/c$:

$$\Delta R = \frac{1}{\left(2B/c\right)} \tag{9}$$

Multiplying the bandwidth by $2/c$ puts it in range frequency units (see [2]). Thus the range resolution in meters is the inverse of the range frequency bandwidth.

3 References

[1] M. A. Richards, *Fundamentals of Radar Signal Processing*. McGraw-Hill, New York, 2005.

[2] M. A. Richards, “Relationship Between Temporal and Spatial Frequency in Radar Imaging”, technical memorandum, July 7, 2007.