

Notes on the Billingsley ICM Model

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1. The Billingsley ICM Model

The Billingsley model for the power spectrum due to *internal clutter motion* (ICM, also called *intrinsic* clutter motion) of windblown radar clutter, which is mentioned on pp. 478-479 of [1], is (see Chapter 6 of [2])

$$P(v) = \underbrace{\frac{\alpha}{\alpha+1} \delta_D(v)}_{\text{DC term}} + \underbrace{\frac{1}{\alpha+1} \cdot \frac{\beta}{2} \exp(-\beta|v|)}_{\text{AC term}} \quad (1)$$

where

- v is velocity in m/s,
- β is the “exponential shape parameter” in units of s/m,
- α is the “DC/AC” power ratio, and
- $\delta_D(\cdot)$ is the Dirac impulse function.

The DC spectral component $\delta_D(v)$ and the AC spectral component $(\beta/2)\exp(-\beta|v|)$ are both separately normalized to unit total power, *i.e.*

$$\begin{aligned} \int_{-\infty}^{\infty} \delta_D(v) dv &= 1 \\ \int_{-\infty}^{\infty} \frac{\beta}{2} \exp(-\beta|v|) dv &= 1 \end{aligned} \quad (2)$$

which also guarantees that the total power spectrum is normalized to unit power,

$$\int_{-\infty}^{\infty} P(v) dv = 1. \quad (3)$$

The model can also be expressed in terms of Doppler frequency instead of velocity using the conversion $F_D = 2v/\lambda$, where λ is the radar wavelength:

$$P(F_D) = \frac{\alpha}{\alpha+1} \delta_D(F_D) + \frac{1}{\alpha+1} \cdot \frac{\lambda\beta}{4} \exp\left(-\frac{\lambda\beta}{2}|F_D|\right) \quad (4)$$

The factor of $\lambda\beta/4$ on the AC term is necessary to maintain the unit power normalization.

Note that the AC term has a two-sided, symmetric exponential decay. However, when plotted in dB (*i.e.*, $10\log_{10}[P(F_D)]$ vs. F_D), the AC term appears as a triangular function. This observation explains why the AC spectral component is sketched as triangular in shape in some references, *e.g.* [3].

Billingsley gives nominal empirical expressions for α and β for general windblown clutter [2]. The DC/AC ratio α is a function of both wind speed and radar frequency,

$$\alpha = 489.9 \cdot w^{-1.55} F_0^{-1.21} \quad (5)$$

where

- w is the wind speed in statute miles per hour, and
- F_0 is the radar carrier frequency in GHz.

The expression for β depends only on windspeed and is given by

$$\beta^{-1} = 0.1048[\log_{10} w + 0.4147] \quad (6)$$

Caution is needed in applying Eq. (6) due to mixed units. Specifically, w is in statute miles per hour, but β is in meters per second.

2. Applicability of the Billingsley ICM Model

Note what α and β do not depend on in this model: the physical type of the clutter. According to Mountcastle,

“Billingsley ... has introduced an ICM model that is in excellent agreement with a wide range of wind-blown clutter environments, and which has gained wide acceptance for empirical work.”

and

“The data are derived from measurements of windblown forest-covered terrain from an instrumentation radar, but are considered to be representative of various types of terrain cover, based on limited observations.”

Billingsley himself states that the model was developed for windblown vegetation. In Section 6.4 of [2], he discusses its application to other terrain types, such as desert, cropland, and rangeland. The implication is that the model is reasonable for use in these terrain types as well,

but with significant adjustments to the DC/AC ratios depending on the clutter type, which would alter the expression (5).

3. Billingsley ICM Model for NLCD Clutter Classes

One specific implementation of the Billingsley ICM model at the Georgia Tech Research Institute (GTRI) [4] adjusts the DC/AC ratio α for clutter type by computing α according to Eq. (5), and then dividing it by a correction factor based on the NLCD data class to form a modified α' according to

$$\alpha' = \frac{\alpha}{k_{NLCD}} \quad (7)$$

where k_{NLCD} is chosen according to the following table (applicable to the NLCD 1999 land categories). The parameter α' is then used in place of α in Eq. (4). The correction factors in Table 1 are rough empirical estimates and should be refined with additional measurements when available.

Table 1. DC/AC Ratio Correction Factor for Billingsley ICM Model

NLCD 1999 Land Class	Class Number	k_{NLCD}	dB Equivalent
Open Water	11	1	0
Perennial Ice/Snow	12	0.00316	-25
Low Intensity Residential	21	0.00316	-25
High Intensity Residential	22	0.00316	-25
Commercial/Industrial/Transportation	23	0.00316	-25
Bare Rock/Sand/Clay	31	0.00316	-25
Quarries/Strip Mines/Gravel Pits	32	0.00316	-25
Transitional	33	0.0316	-15
Deciduous Forest	41	1.0	0
Evergreen Forest	42	1.0	0
Mixed Forest	43	1.0	0
Shrubland	51	0.316	-5
Orchards/Vineyards/Other	61	0.316	-5
Grasslands/Herbaceous	71	0.316	-5
Pasture/Hay	81	0.316	-5
Row Crops	82	0.316	-5
Small Grains	83	0.316	-5
Fallow	84	0.0316	-15
Urban/Recreational Grasses	85	0.316	-5
Woody Wetlands	91	1	0
Emergent Herbaceous Wetlands	92	0.316	-5

A MATLAB function `clut_nlcd2icm_billingsley.m` was made available by GTRI. This function implements the lookup of k_{NLCD} based on the NLCD 1999 class number. The version included here is slightly modified from the original in order to fix a bug, based on discussions with GTRI [5].

4. Generation of Random Processes Conforming to the Billingsley ICM Model

Slow-time data sequences that have a Billingsley power spectrum can be generated by passing a white random process through an appropriate linear shift-invariant filter to obtain the AC term, and adding an appropriate constant to the resulting data to create the DC term with the desired DC/AC ratio α . Mountcastle [3] describes two specific example IIR filters that are suitable for generating the AC data, and shows how to scale their parameters to produce the desired value of β .

Note that if the filter input is a white Gaussian random process, as is expected to normally be the case, then the output will be colored but still Gaussian. The Gaussian statistics imply that a many-scatterer model is effectively assumed for the AC component of the clutter signal.

5. Generalized Billingsley ICM Model

It is easy to imagine a generalization of the Billingsley ICM model that uses the DC/AC component structure of Eq. (1) or (4), but replaces the exponential AC component with a different AC power spectrum component. One obvious choice, reasonably well supported by data [2], is a Gaussian AC component. In this case, the generalized Billingsley model would become, in both velocity and Doppler frequency forms,

$$\begin{aligned} P(v) &= \frac{\alpha}{\alpha+1} \delta_D(v) + \frac{1}{\alpha+1} \cdot \frac{1}{\sigma_G \sqrt{2\pi}} \exp\left(-v^2/2\sigma_v^2\right) \\ P(F_D) &= \frac{\alpha}{\alpha+1} \delta_D(F_D) + \frac{1}{\alpha+1} \cdot \frac{1}{\sigma_{F_D} \sqrt{2\pi}} \exp\left(-F_D^2/2\sigma_{F_D}^2\right) \end{aligned} \quad (8)$$

where σ_v is the standard deviation in m/s and σ_{F_D} is the standard deviation in s^{-1} . The two are related according to $\sigma_{F_D} = (\lambda/2)\sigma_v$.

In general, the exponential model will have longer power spectrum “tails” than the Gaussian model (the power law models cited in some literature have even wider tails) [2]. The table on the next page, obtained courtesy of [4], provides values of the standard deviation σ_v for the Gaussian component in units of m/s. A MATLAB lookup table for this data is provided in the

function `clut_nlcd2icm_gaussian.m`. Note that that function actually squares σ_v so as to return the variance in (m/s^2).

GTRI evidently uses a pure Gaussian (no DC component) spectrum model when the Gaussian model is selected [5]. This is accomplished by just setting $\alpha = 0$.

Table 2. Standard Deviation of Gaussian Component for Billingsley-Gaussian ICM Model

NLCD 1999 Land Class	Class Number	σ_G (m/s)
Open Water	11	0
Perennial Ice/Snow	12	0
Low Intensity Residential	21	0
High Intensity Residential	22	0
Commercial/Industrial/Transportation	23	0
Bare Rock/Sand/Clay	31	0
Quarries/Strip Mines/Gravel Pits	32	0
Transitional	33	0
Deciduous Forest	41	1.3
Evergreen Forest	42	1.3
Mixed Forest	43	1.3
Shrubland	51	.3
Orchards/Vineyards/Other	61	.1
Grasslands/Herbaceous	71	.3
Pasture/Hay	81	.1
Row Crops	82	.6
Small Grains	83	.3
Fallow	84	0
Urban/Recreational Grasses	85	.1
Woody Wetlands	91	.3
Emergent Herbaceous Wetlands	92	.3

6. Code for Model Parameter Lookup

Two MATLAB functions are included in the following pages, both obtained courtesy of [4]. `clut_nlcd2icm_billingsley.m` implements the table lookup of k_{NLCD} . It is slightly modified from the original in order to correct a bug [5]. `clut_nlcd2icm_gaussian.m` implements the lookup of σ_G .

7. References:

- [1] M. A. Richards, *Fundamentals of Radar Signal Processing*. (McGraw-Hill, New York, 2005.)

- [2] J. B. Billingsley, *Low-Angle Radar Clutter*. (SciTech Publishing and William Andrews Publishing, 2002.)
- [3] P. D. Mountcastle, "New Implementation of the Billingsley Clutter Model for GMTI Data Cube Generation," *Proceedings IEEE 2004 Radar Conference*, pp. 389-401, April 2004.
- [4] R. Hersey, GTRI, personal communication, January 2009.
- [5] G. A. Showman, GTRI, personal communication, February 2009.

clut_nlcd2icm_billingsley.m

```
% file name:      clut_nlcd2icm_billingsley.m
% description:    Reclassify NLCD land characterization data to icm model
% created by:     djz, GTRI/SEAL/RSD/SPB
% created on:     9/26/02
```

```
function [ acdc_mod ] = clut_nlcd2icm_billingsley(range_nlcd)
```

```
%      Water
%      11 Open Water
%      12 Perennial Ice/Snow
%      Developed
%      21 Low Intensity Residential
%      22 High Intensity Residential
%      23 Commercial/Industrial/Transportation
%      Barren
%      31 Bare Rock/Sand/Clay
%      32 Quarries/Strip Mines/Gravel Pits
%      33 Transitional
%      Vegetated; Natural Forested Upland
%      41 Deciduous Forest
%      42 Evergreen Forest
%      43 Mixed Forest
%      Shrubland
%      51 Shrubland
%      Non-natural Woody
%      61 Orchards/Vineyards/Other
%      Herbaceous Upland
%      71 Grasslands/Herbaceous
%      Herbaceous Planted/Cultivated
%      81 Pasture/Hay
%      82 Row Crops
%      83 Small Grains
%      84 Fallow
%      85 Urban/Recreational Grasses
%      Wetlands
%      91 Woody Wetlands
%      92 Emergent Herbaceous Wetlands
```

```
% icm reclass for delmarva scenario (reclass land use to icm s.d. for
gaussian model)
```

```
icm_reclass_base= [
```

```
11 1; % 0 dB
12 0.00316; % -25 dB
21 0.00316; % -25 dB
22 0.00316; % -25 dB
23 0.00316; % -25 dB
31 0.00316; % -25 dB
32 0.00316; % -25 dB
33 0.0316; % -15 dB
41 1.0; % 0 dB
42 1.0; % 0 dB
```

```
43 1.0; % 0 dB
51 0.316; % -5 dB
61 0.316; % -5 dB
71 0.316; % -5 dB
81 0.316; % -5 dB
82 0.316; % -5 dB
83 0.316; % -5 dB
84 0.0316; % -15 dB
85 0.316; % -5 dB
91 1; % 0 dB
92 0.316; % -5 dB
];

if nargin < 1

    figure,stem(icm_reclass_base(:,1),icm_reclass_base(:,2)),grid
    xlabel('NLCD Type'),ylabel('AC-DC ICM Power Modification')

else

    % Input check.
    range_nlcd = round(range_nlcd); % Make sure input is integer.
    range_nlcd(find(range_nlcd < 0)) = 0; % Set values less than 0 to 0.
    range_nlcd(find(range_nlcd > 92)) = 0; % Set values greater than 92 to 0.

    % Add 1 to account for NLCD type 0
    icm_reclass(icm_reclass_base(:,1)+1) = icm_reclass_base(:,2);
    acdc_mod = (icm_reclass(range_nlcd+1));

end
```


clut_nlcd2icm_gaussian.m

```
% file name:      clut_nlcd2icm_gaussian.m
% description:    Reclassify NLCD land characterization data to icm model
% created by:     djz, GTRI/SEAL/RSD/SPB
% created on:     9/26/02
```

```
function [ range_icm_sd ] = clut_nlcd2icm_gaussian(range_nlcd)
```

```
%      Water
%      11 Open Water
%      12 Perennial Ice/Snow
%      Developed
%      21 Low Intensity Residential
%      22 High Intensity Residential
%      23 Commercial/Industrial/Transportation
%      Barren
%      31 Bare Rock/Sand/Clay
%      32 Quarries/Strip Mines/Gravel Pits
%      33 Transitional
%      Vegetated; Natural Forested Upland
%      41 Deciduous Forest
%      42 Evergreen Forest
%      43 Mixed Forest
%      Shrubland
%      51 Shrubland
%      Non-natural Woody
%      61 Orchards/Vineyards/Other
%      Herbaceous Upland
%      71 Grasslands/Herbaceous
%      Herbaceous Planted/Cultivated
%      81 Pasture/Hay
%      82 Row Crops
%      83 Small Grains
%      84 Fallow
%      85 Urban/Recreational Grasses
%      Wetlands
%      91 Woody Wetlands
%      92 Emergent Herbaceous Wetlands
```

```
% icm reclass for delmarva scenario (reclass land use to icm s.d. for gaussian model)
```

```
icm_reclass_base= [
0      0;
11     0;
12     0;
21     0;
22     0;
23     0;
31     0;
32     0;
33     0;
41    1.3; %.22;
```

```
42 1.3;  %.22;
43 1.3;  %.22;
51 .3;   %.1;
61 .1;   %.03;
71 .3;   %.1;
81 .1;   %.02;
82 .6;   %.19;
83 .3;   %.08;
84 0;
85 .1;   %.02;
91 .3;   %.07;
92 .3   %.07
];

icm_reclass_base(:,2) = icm_reclass_base(:,2).^2;

if nargin < 1

    figure,stem(icm_reclass_base(:,1),icm_reclass_base(:,2)),grid
    xlabel('NLCD Type'),ylabel('ICM Variance, m^2/s^2')

else

    % Input check.
    range_nlcd = round(range_nlcd);           % Make sure input is integer.
    range_nlcd(find(range_nlcd < 0)) = 0;     % Set values less than 0 to 0.
    range_nlcd(find(range_nlcd > 92)) = 0;    % Set values greater than 92 to 0.

    % Add 1 to account for NLCD type 0
    icm_reclass(icm_reclass_base(:,1)+1) = icm_reclass_base(:,2);
    range_icm_sd = (icm_reclass(range_nlcd+1));

end
```