Generating Swerling Random Sequences

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January 21, 1999
Revised December 15, 2008

1 BACKGROUND

The Swerling models are models of the probability density function (pdf) and time correlation properties of the radar backscatter from a complex target [1]. Developed in the early days of radar, the Swerling models apply to a finite group of pulses. They were developed with the model of a rotating surveillance radar in mind. As the radar beam sweeps past a target (a single *scan*), it collects echoes from that target in the appropriate range bin for several pulses. Once the beam moves past the target, no more echoes are received until the next scan, when the beam has swung back around to the target position again; another group of several pulses is then received. Detection is assumed to be attempted using all of the pulses from a single scan. Thus, the joint statistics of a group of target echo samples from contiguous pulses of a single scan are of interest.

The Swerling models are formed from the four combinations of two probability density functions (pdf's) for the individual echo powers and two assumptions regarding the decorrelation time, or independence, of pulses within a single scan according to the following table:

SWERLING TARGET MODELS

probability density function of power	***decorrelation***	
	scan-to-scan	pulse-to-pulse
exponential	**1**	**2**
chi-square, degree 4	**3**	**4**

The individual echo powers (proportional to radar cross section) are assumed to exhibit either an exponential pdf (Swerling 1 and 2) or a 4^{th}-degree chi-square pdf (Swerling 3 and 4). The corresponding voltage distributions (square root of power) are the Rayleigh and the 4^{th}-degree chi distributions. The Rayleigh voltage/exponential power pdf, which is obtained from a law of large numbers argument, is appropriate for a target composed of a large number of approximately equal-strength scatterers, with no one scatterer dominant. It is often applied to large (with respect to wavelength), complex targets, especially when viewed over changing aspect angles.

The 4th-degree chi voltage/4th-degree chi-square power pdf is an approximation to the pdf obtained in the case of a large number of equal strength scatterers plus a single, steady dominant scatterer, with the power of the dominant scatterer equal to $1+\sqrt{2}$ times the total power of all the smaller scatterers.[1] The exact distribution for this case is the Rice or Rician distribution, which can model any ratio of the dominant to lesser scatterers. However, the Swerling approximation is well-entrenched, partly because it is more analytically tractable.

Note that the voltage models produce only non-negative values. It is assumed that what is being modeled is the output of a linear detector, *i.e.* the magnitude of the real or complex video receiver voltage. The power models correspond to a square-law detector.

Concerning decorrelation, the term *scan-to-scan decorrelation* implies that all the echoes within a given scan have the same value, drawn from the appropriate pdf. All of the pulses on the next scan are again equal to one another, but not to those in the previous scan. Instead, they have a new, independent value drawn from the appropriate pdf. In *pulse-to-pulse decorrelation* each individual echo power sample is a new, independent random variable drawn from the appropriate pdf. Physically, decorrelation is typically assumed to be caused by changing radar-target aspect angle from one measurement to the next. It can also be forced through the use of *frequency agility* [1].

Reference is sometimes made to a “Swerling 0” or “Swerling 5” model. This is the case of a single, nonfluctuating scatterer and thus does not require the generation of random variables.

2 ALGORITHMS

The only issue in generating Swerling random sequences is the generation of independent random variables having the various probability density functions required for voltage or power samples. Decorrelation models are implemented by either using one random variable from the desired pdf for all N samples in a scan (scan-to-scan decorrelation, *i.e.* the Swerling 1 or 3 models), or generating separate random variables of the desired pdf for each of the N samples (pulse-to-pulse decorrelation, *i.e.* the Swerling 2 or 4 models).

2.1 EXPONENTIAL DISTRIBUTION

The exponential probability density function of mean μ is given by [2]

$$p_x(x) = \begin{cases} \dfrac{1}{\mu} e^{-x/\mu}, & x \geq 0 \\ 0, & x < 0 \end{cases} \tag{1}$$

[1] The particular dominant/small scatterer ratio of $1+\sqrt{2}$ is the ratio that causes the first two moments (mean and variance) of the Rice distribution to match the chi-square distribution.

The standard deviation of this exponential random variable is μ^2.

A very simple way to generate exponential-distributed random variables x from uniform [0,1) rv's u is by the following transformation [3]:

$$x = -\mu \ln u \qquad (2)$$

Figure 1 is a histogram obtained by applying this transformation with $\mu = 1$ to a vector of 100,000 uniform rv's generated in MATLAB. Also shown is the theoretical probability density function. The theoretical standard deviation for this distribution is also 1. The sample mean and standard deviation were 1.0027 and 1.0019; the differences are about 0.2% in the standard deviation and 0.3% in the mean.

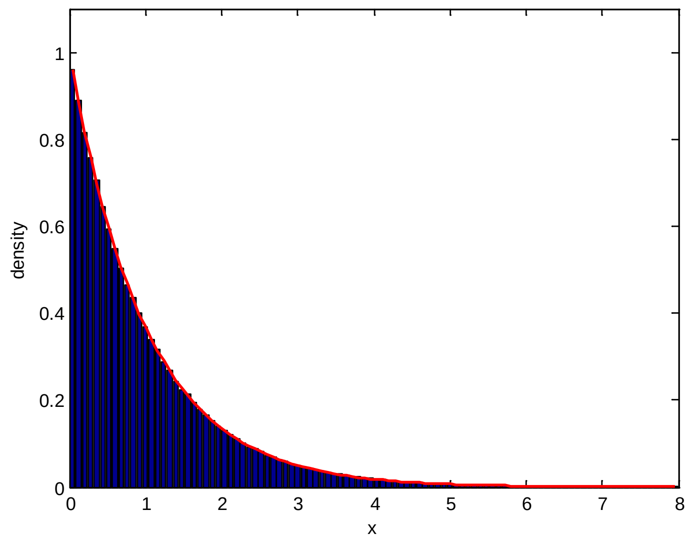

Figure 1. 100-bin histogram of a unit-mean exponential sample sequence generated in MATLAB using the native MATLAB uniform random number generator and Eqn. (2). The corresponding theoretical pdf of Eqn. (1) is also shown.

2.2 RAYLEIGH DISTRIBUTION

The Rayleigh probability density function of mean μ is given by [2]

$$p_x(x) = \begin{cases} \dfrac{\pi x}{2\mu^2} e^{-\pi x^2/4\mu^2}, & x \ge 0 \\ 0, & x < 0 \end{cases} \qquad (3)$$

The standard deviation of this Rayleigh random variable is $\mu\sqrt{(4-\pi)/\pi} = 0.5227\mu$.

A simple way to generate Rayleigh-distributed random variables x from uniform [0,1) rv's u is by the following transformation [3]:

$$x = \sqrt{-\frac{4\mu^2}{\pi}\ln u} \tag{4}$$

The logarithm converts the uniform distribution to an exponential distribution; the scale factor adjusts the mean; and the square root converts the exponential distribution to a Rayleigh.

Figure 2 is a histogram obtained by applying this transformation with $\mu = 1$ to a vector of 100,000 uniform rv's generated in MATLAB. Also shown is the theoretical probability density function. The theoretical standard deviation for this distribution is 0.522723. The sample mean and standard deviation were 1.0002 and 0.52211; the differences are about 0.1% in the standard deviation and 0.2% in the mean.

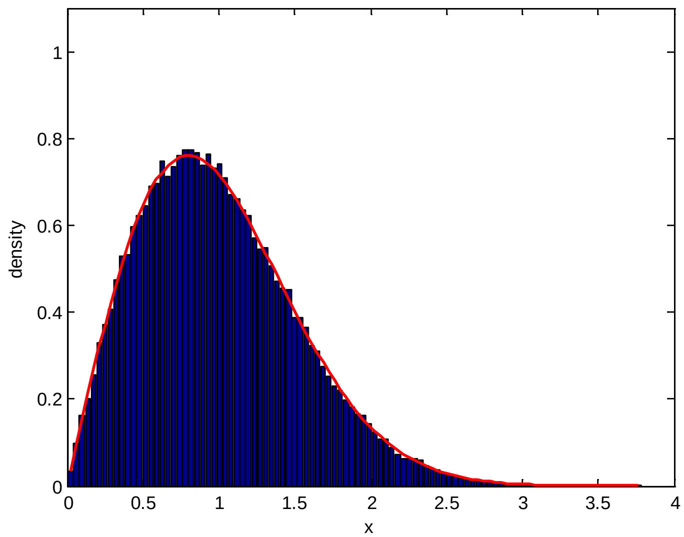

Figure 2. 100-bin histogram of a unit-mean Rayleigh sample sequence generated in MATLAB using the native MATLAB uniform random number generator and Eqn. (4). The corresponding theoretical pdf of Eqn. (3) is also shown.

2.3 4TH-DEGREE CHI-SQUARE DISTRIBUTION

The 4th-degree chi-square probability density function of mean μ is given by [2]

$$p_x(x) = \begin{cases} \frac{4x}{\mu^2} e^{-2x/\mu}, & x \ge 0 \\ 0, & x < 0 \end{cases} \tag{5}$$

The variance σ^2 of this random variable is $\mu^2/2$.

A simple way to generate 4th-degree chi-square random variables x is by the following transformation [3]:

$$x = -\frac{\mu}{2}\ln\left(u_1 u_2\right) \tag{6}$$

where u_1 and u_2 are independent uniform (0,1] rvs. Notice the similarity to Eqn. (2).

Figure 3 is a histogram of 100,000 4^{th}-degree chi samples obtained by applying Eqn. (6) with $\mu = 1$ to 200,000 uniform rv's generated in MATLAB. Also shown is the theoretical probability density function. The theoretical standard deviation for this distribution is 0.7071. The sample mean and standard deviation were 1.003 and 0.7114; the differences are about 0.3% in the mean and about 0.6% in the standard deviation.

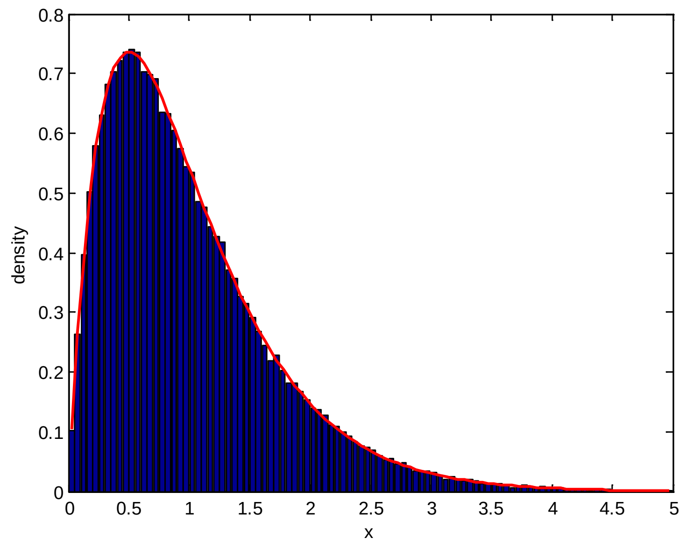

Figure 3. 100-bin histogram of a unit-mean 4^{th}-degree chi-square sample sequence generated in MATLAB using the native MATLAB uniform random number generator and the method of Eqn. (6). The corresponding theoretical pdf of Eqn. (5) is also shown.

2.4 4^{TH}-DEGREE CHI DISTRIBUTION

The 4^{th}-degree chi distribution having mean μ is given by [2]

$$p_x(x) = \begin{cases} \dfrac{\alpha^4}{2\mu^4} x^3 e^{-\alpha^2 x^2 / 2\mu^2}, & x \geq 0 \\ 0, & x < 0 \end{cases} \tag{7}$$

where $\alpha \equiv \sqrt{2} \cdot \Gamma(5/2)/\Gamma(2) \approx 1.88$. The variance σ^2 of this random variable is $(\mu/\alpha)^2\left(4-\alpha^2\right)$.

Just as a Rayleigh rv could be obtained as the square root of an exponential rv with appropriate scaling to get the desired mean, a simple way to generate 4th-degree chi-distributed random variables x from uniform [0,1) rv's u is by the following transformation:

$$x = \sqrt{-\frac{2\mu^2}{\alpha^2}\ln\left(u_1 u_2\right)} \qquad (8)$$

where u_1 and u_2 are independent uniform (0,1] random variables and α is as described in the previous paragraph. Notice the similarity to Eqn. (4).

Figure 4 is a histogram of 100,000 4th-degree chi samples obtained by applying Eqn. (6) with $\mu = 1$ to 200,000 uniform rv's generated in MATLAB. Also shown is the theoretical probability density function. The theoretical standard deviation for this distribution is 0.3630. The sample mean and standard deviation were 0.9992 and 0.3616; the differences are about 0.1% in the mean and about 0.4% in the standard deviation.

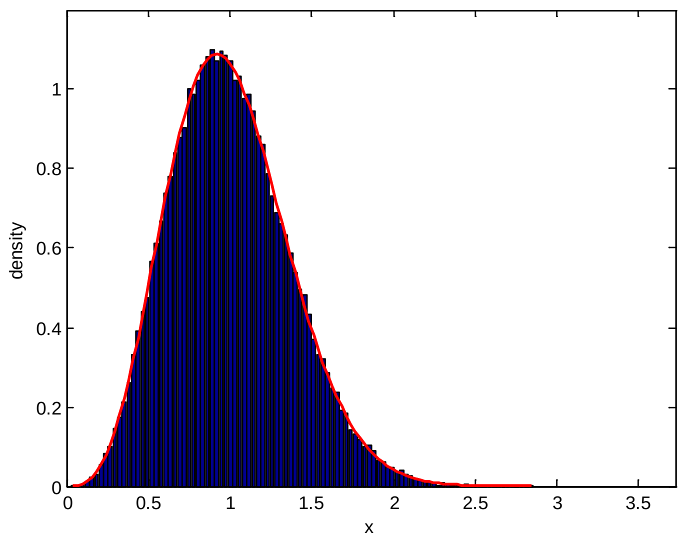

Figure 4. 100-bin histogram of a unit-mean 4th-degree chi sample sequence generated in MATLAB using the native MATLAB uniform random number generator and the method of Eqn. (8). The corresponding theoretical pdf of Eqn. (7) is also shown.

3 References

[1] M. A. Richards, *Fundamentals of Radar Signal Processing*. (McGraw-Hill, New York, 2005.)

[2] A. Papoulis and S. U. Pillai, *Probability, Random Variables, and Stochastic Processes*, fourth edition. (McGraw-Hill, New York, 2002.)

[3] J. E. Gentle, *Random Number Generation and Monte Carlo Methods*, 2nd ed. (Springer, New York, 2003.)